NCB Wind Waves and Boat Wakes

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This is an outline for calculating wave activity along a section of shoreline in order to help characterize the amount of energy the shoreline may experience. It is not currently intended for engineering design. Questions? David Christian, dchristi@sjrwmd.com, 386-329-2503.

Wind Waves

The wind generated waves are calculated in the waves.f program following the methods in the Shore Protection Manual (1984), Coastal Engineering Manuals (2002 and 2006), Weggel and Douglass (1985), and Roland and Douglass (2005).

Assumptions that were made:

1. The wind data was gotten from the Daytona Beach International Airport, so it is assumed that it was measured 10 m off the ground.
2. The fetch is taken to be a straight line distance in the direction that the wind blows across the water.
3. Effective fetch is not calculated. This follows CEM (2002, p II-2-45) and SPM (1984, p 3-51).
4. Since the wind data is from the airport, we can assume it was taken over land.
5. Assume that the fetch is less than 16 km, this is reasonable in the NCB. This allows us to convert from wind over land to wind over water using a multiplier of $R_L = 1.2$. (CEM, 2002, p II-2-36), (Weggel and Douglass, 1985, p 9)
6. Also, if we assume a fetch less than 16 km, we do not need to worry about adding in a stability correction, R_T (CEM, 2002, p II-2-36)
7. If the fetch happened to be greater than 16 km, then a multiplier of 1.1 needs to be used for a stability correction (SPM, 1984, p 3-30). 1.1 would be used since we do not know the temperatures.
8. We assume that the waves we are dealing with are fetch limited.
9. We assumed the wind directions went clockwise, starting with 0 degree wind being out of the north.
10. 0 and 360 degree wind were grouped together.
11. All 0 m/s winds in the 0 degree direction were removed since this appeared to be a calm wind time and I did not want to throw off the number of times the wind came from that direction.

Equations used:

Although not used for the NCB, the following two equations should be used if adjustments need to be made for the measured wind data.

If the wind speed is not measured at 10 m above the ground, it needs to be adjusted to the wind speed at 10 m using equation 1.

$$U(10) = U(z)\left(\frac{10}{z}\right)^{1/7} \qquad (1)$$

where:
U(10)= The wind speed at 10 m above the ground
U(z) = The wind speed at *z* meters above the ground (*z*<20m to use this equation)

We assume here that the wind data we have are hourly wind speeds. If we have wind speeds averaged over other time periods, or want to get wind speeds averaged over another time period, use equation 2a or 2b.

$$\frac{U_t}{U_{3,600}} = 1.277 + 0.296 * \tanh\left(0.9 * \log_{10}\frac{45}{t}\right) \qquad \text{(2a)} \quad \text{For } t < 3{,}600 \text{ s}$$

$$\frac{U_t}{U_{3,600}} = -0.15 * \log_{10} t + 1.5334 \qquad \text{(2b)} \quad \text{For } 3{,}600 < t < 36{,}000 \text{ s}$$

$U_{3,600}$= 1-hr wind speed (m/s)
U_t= wind speed over time t(s)

The wind over land was read in and first converted to wind over water using equation 3.

$$U_w = U_L * R_L \qquad (3)$$

Where:
U_w = wind speed over water (m/s)
U_L = wind speed over land at 10m (m/s)
R_L = Factor to convert from over land to over water wind speeds, 1.2 for fetch<16km. For fetch > 16km, Figure 3-15 in SPM (1984) or Figure II-2-7 in SPM (2002) should be used to find R_L.

If the fetch is greater than 16 km (which we don't really need to worry about in the NCB), then we need to adjust for stability. This comes from the difference in temperature between the air and the water. The equation is equation 4. If the air and water temperatures are unknown, then use R_T = 1.1. If the temperatures are known, then use Figure 3-14 in SPM (1984) or Figure II-2-8 in CEM(2002). If just general knowledge is known about the temperatures, then the following guide from CEM(2002) can be used:

If air is warmer than water, then the condition is stable and $R_T = 0.9$
If air and water temperatures are equal, then the condition is neutral and $R_T = 1.0$
If air is cooler than water, then the condition is unstable and $R_T = 1.1$

$U = R_T\,U(10)$ (4)

The equations from the SPM for wave growth use a wind-stress factor, U_A, which is an adjusted wind speed. Once the wind speed is adjusted to being over water and for stability (if necessary), it needs to be converted into U_A before being used in the wave equations.

$U_A = 0.71 * U^{1.23}$ (5) for U in m/s

For shallow water wave prediction, the following equations should be used. These are also in curve form in Figures 3-27 through 3-36 in SPM (1984). These equations use dimensionless parameters $\frac{gH}{U_A^2}$, $\frac{gd}{U_A^2}$, $\frac{gF}{U_A^2}$, and $\frac{gT}{U_A}$, so they are okay for metric or English units. Equation 6 calculates wave height while Equation 7 calculates wave period.

$$\frac{gH}{U_A^2} = 0.283 \tanh\left[0.530\left(\frac{gd}{U_A^2}\right)^{3/4}\right] \tanh\left\{\frac{0.00565\left(\frac{gF}{U_A^2}\right)^{1/2}}{\tanh\left[0.530\left(\frac{gd}{U_A^2}\right)^{3/4}\right]}\right\} \qquad (6)$$

$$\frac{gT}{U_A} = 7.54 \tanh\left[0.833\left(\frac{gd}{U_A^2}\right)^{3/8}\right] \tanh\left\{\frac{0.0379\left(\frac{gF}{U_A^2}\right)^{1/3}}{\tanh\left[0.833\left(\frac{gd}{U_A^2}\right)^{3/8}\right]}\right\} \qquad (7)$$

Once the period is found, then the minimum duration of wind to reach the fully arisen sea conditions, *t*, can be found using Equation 8.

$$\frac{gt}{U_A} = 5.37 \times 10^2 \left(\frac{gT}{U_A}\right)^{7/3} \qquad (8)$$

The other option, as recommended in CEM 2006 is to use Deep Water wave equations for shallow water. U_w and U are found as before, but instead of calculating U_A , the friction velocity (u_*) is calculated instead using Equations 9, 10, and 11.

$$C_D = \frac{u_*^2}{U^2} \qquad \text{(9a)}$$

rearranged to:

$$u_* = U\sqrt{C_D} \qquad \text{(9b)}$$

$$C_D = 0.001(1.1 + 0.035U) \qquad \text{(10)}$$

put 10 into 9b:

$$u_* = U\sqrt{0.001(1.1 + 0.035U)} \qquad \text{(11)}$$

where:

C_D = drag coefficient

U here is the same as U_W

u_* is then used to calculate wave height and period using equations 12 and 13.

$$\frac{gH_{m_0}}{u_*^2} = 4.13\times10^{-2} * \left(\frac{gF}{u_*^2}\right)^{\frac{1}{2}} \qquad \text{(12)}$$

$$\frac{gT_P}{u_*} = 0.751 * \left(\frac{gF}{u_*^2}\right)^{\frac{1}{3}} \qquad \text{(13)}$$

where:

H_{m_0} = energy-based significant wave height

Using the deep water method, we have different equations that are used for fully developed wave conditions. These are shown in equations 14 and 15.

$$\frac{gH_{m_*}}{u_*^2} = 2.115\times10^2 \qquad \text{(14)}$$

$$\frac{gT_P}{u_*} = 2.398\times10^2 \qquad \text{(15)}$$

Equation 16 should be used to find the limiting wave period for the deep water equations.

$$T_P \approx 9.78\left(\frac{d}{g}\right)^{\frac{1}{2}} \qquad (16)$$

In order to use the deep water wave equations in shallow water, CEM 2006 recommends the following:

- Compare the period calculated in Equation 13 to the period calculated with Equation 16. If the Equation 13 value is greater, then reduce it to the same as from Equation 16. The dimensionless fetch that would correspond with to this wave period can be used for fetch in the wave height equation to calculate the wave height.
- If the wave height is greater than 0.6 times the depth, then the wave height should be decreased to 0.6 times the depth.

A test of both of these methods was done near Ormond Beach on the Halifax River. Figure 1 shows the test site location as well as the compass laid overtop.

Figure 1: Test site at Ormond Beach.

The length of the fetch along each 10^0 was found as in Figure 2.

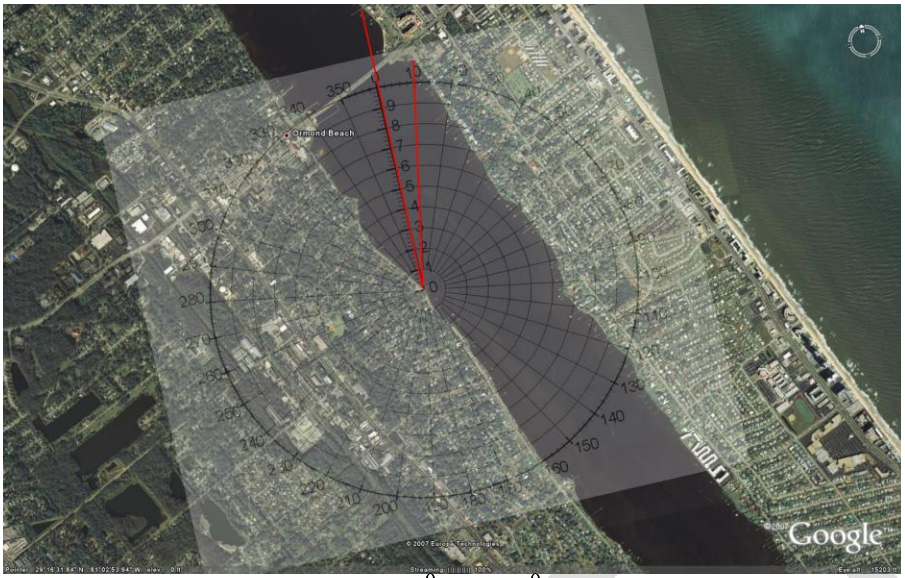

Figure 2: Fetch lines along the 0^0 and 10^0 lines of the compass.

If the depths were known, the average depth along each fetch would have also been found. We used 2 m as the average depth along all of the fetch lines for the shallow water equations where depth is a needed input.

The method followed to find the wave heights at the location is very similar to the method outlined in Roland and Douglass (2005). The wind speeds in each 10^0 direction were gotten from data at Daytona Beach International Airport (EarthInfo NCDC Surface Airways data). This wind data is to be used for the whole NCB. Since we're looking at long term trends, using the one site is sufficient. The wind speeds were then rounded and grouped into whole number wind speeds (e.g. 0 m/s, 1 m/s, 2 m/s, …., 26 m/s) . The percent of wind coming from each direction was plotted on a wind rose as shown in Figure 3.

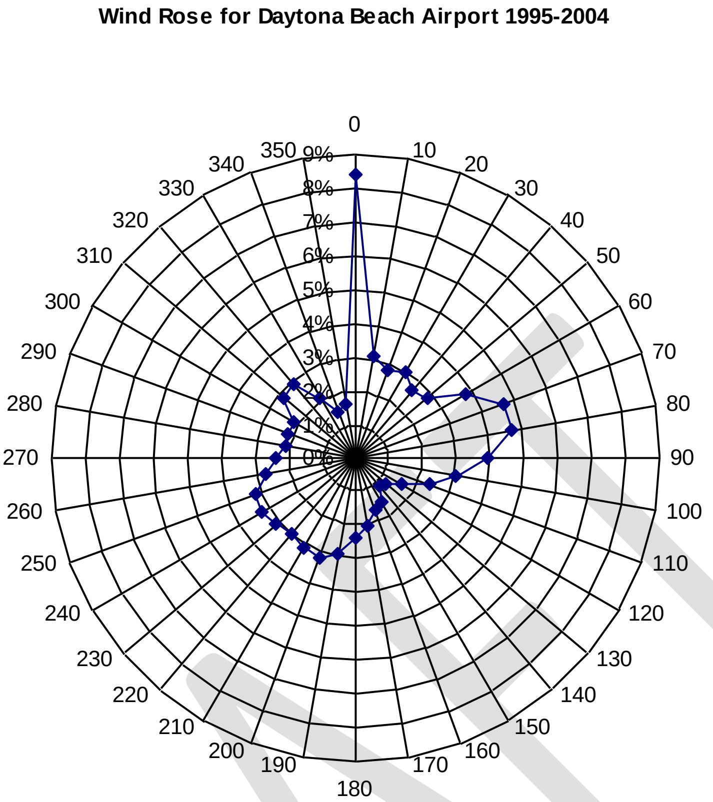

Figure 3: Wind rose for Daytona Beach International Airport for June 1995 - December 2004.

The wind rose was also found for the highest 20% of the wind speeds as shown in Figure 4.

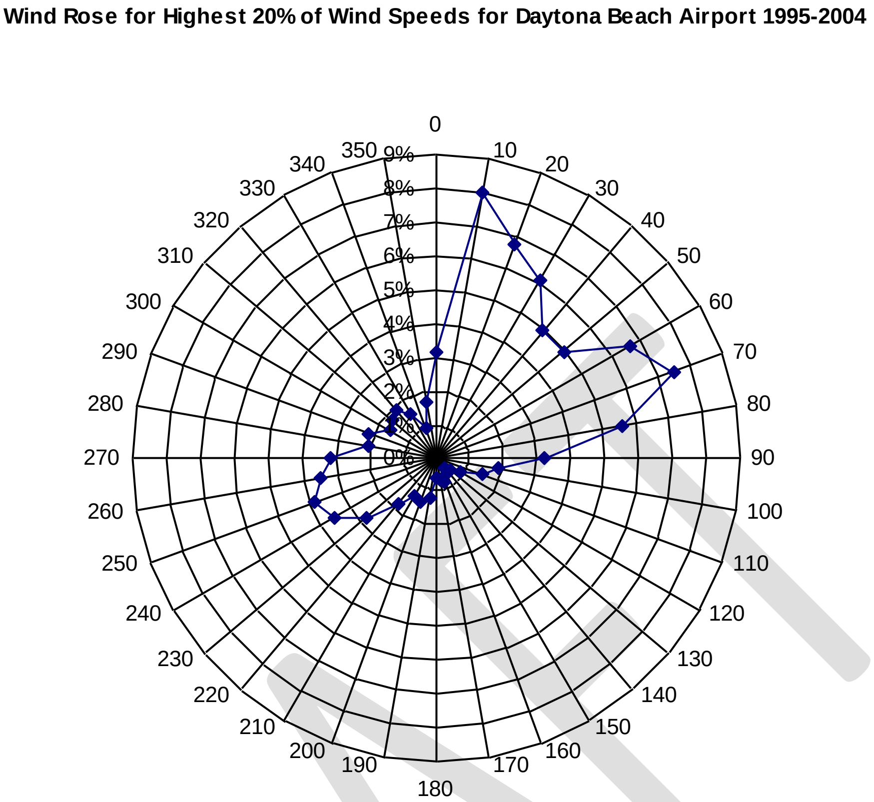

Figure 4: Wind rose for Daytona Beach International Airport for June 1995 - December 2004 for the highest 20% of wind speeds.

Once the wind speeds are grouped, a FORTRAN program reads in the fetch length, and average depth along the fetch (for shallow water calculations). It then reads in the different wind speeds along each 10^0 fetch and calculates a significant wave height for each wind speed along each fetch. The program then groups each wave height into 0.05 m groups (e.g. 0-0.05, 0.051-0.10, …., 1.001-1.05 m). The number of wave occurrences in each group is found and then the percentage of waves with wave heights less than the wave group is found. The results are then plotted to see the distribution. Figure 5 shows a comparison of these distributions using the shallow water and deep water equations.

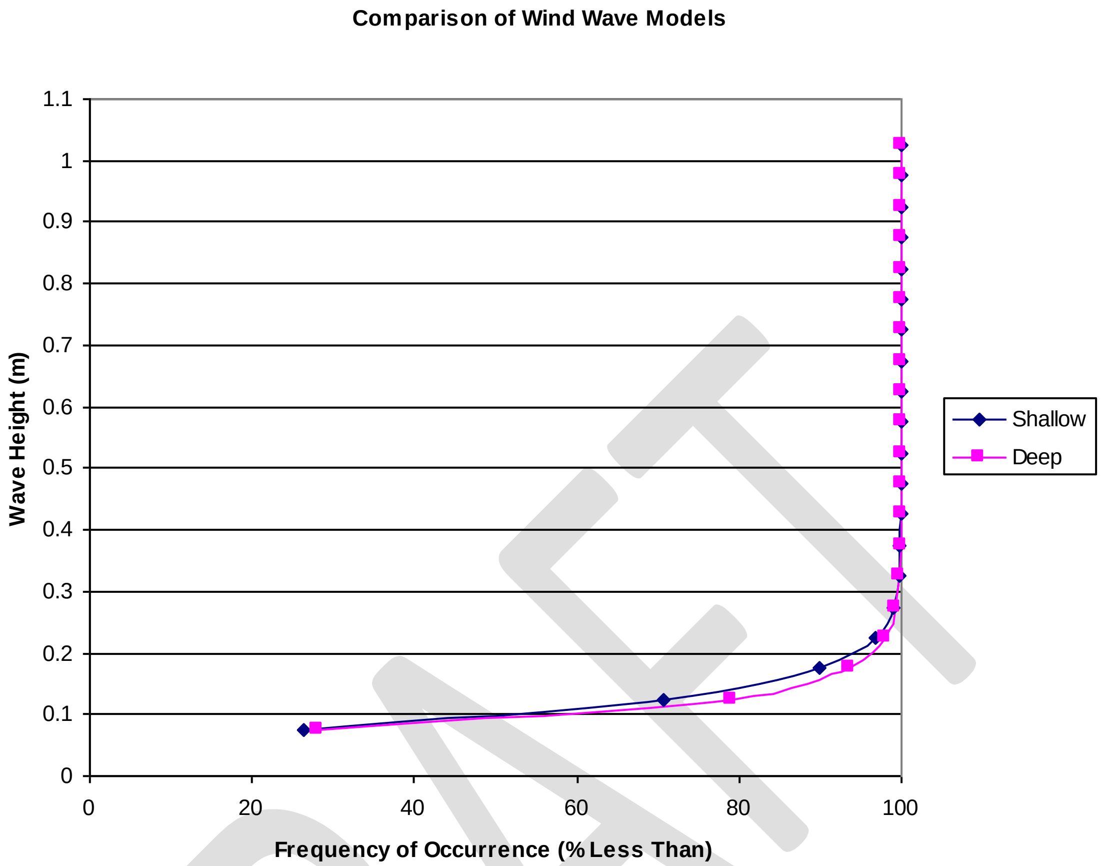

Figure 5: Frequency of occurrence of wave heights near Ormond Beach using both shallow water and deep water equations.

With both the shallow and deep water equations, over 90% of the waves are below 0.2 m. Tables 1-3 show the comparison of computed wave heights using shallow and deep water equations for the minimum, mean, and maximum fetches for the site near Ormond Beach.

Table 1: Calculated wave heights for the Ormond Beach site for the minimum fetch (708 m).

Wind Speed (m/s)	H (m) Shallow Water Equations	H (m) Deep Water Equations
1	0.011	0.014
2	0.028	0.029
3	0.046	0.044
4	0.065	0.060
5	0.086	0.076
6	0.107	0.093
7	0.128	0.110
8	0.151	0.128
9	0.173	0.146
10	0.196	0.164
11	0.219	0.183
12	0.243	0.202
13	0.267	0.222
14	0.290	0.242
15	0.314	0.263
16	0.338	0.284
17	0.363	0.305
18	0.387	0.327
19	0.411	0.349
20	0.435	0.371
21	0.460	0.394
22	0.484	0.417
23	0.508	0.440
24	0.532	0.464
25	0.557	0.488
26	0.581	0.513

Table 2: Calculated wave heights for the Ormond Beach site for the mean fetch (1464 m).

Wind Speed (m/s)	H (m) Shallow Water Equations	H (m) Deep Water Equations
1	0.015	0.020
2	0.039	0.042
3	0.065	0.064
4	0.092	0.086
5	0.121	0.110
6	0.150	0.134
7	0.179	0.158
8	0.209	0.184
9	0.239	0.210
10	0.270	0.236
11	0.300	0.263
12	0.331	0.291
13	0.361	0.319
14	0.392	0.348
15	0.422	0.378
16	0.452	0.408
17	0.482	0.439
18	0.512	0.470
19	0.541	0.501
20	0.570	0.534
21	0.600	0.566
22	0.628	0.600
23	0.657	0.633
24	0.685	0.667
25	0.713	0.702
26	0.741	0.737

Table 3: Calculated wave heights for the Ormond Beach site for the maximum fetch (5545 m).

Wind Speed (m/s)	H (m) Shallow Water Equations	H (m) Deep Water Equations
1	0.021	0.040
2	0.069	0.081
3	0.117	0.124
4	0.165	0.168
5	0.211	0.213
6	0.257	0.260
7	0.302	0.308
8	0.346	0.357
9	0.388	0.408
10	0.429	0.460
11	0.469	0.513
12	0.508	0.567
13	0.546	0.622
14	0.582	0.678
15	0.617	0.736
16	0.652	0.794
17	0.685	0.854
18	0.718	0.914
19	0.749	0.976
20	0.780	1.038
21	0.810	1.102
22	0.839	1.167
23	0.868	1.200
24	0.896	1.200
25	0.923	1.200
26	0.950	1.200

For the shorter fetches, both sets of equations are pretty close to each other, with the shallow water equations generally predicting slightly higher waves. For the maximum fetch, however, the deep water equations predicted higher waves, with the higher wind speeds showing a considerable difference between the two methods. As Figure 6 shows, the wind speed is very rarely greater than 12 m/s at Daytona beach Airport. The higher wind speeds used here are purely for comparison's sake. In cases such as this site, where most of the fetches are relatively short, the differences shown between the two probably don't matter too much. If the depth of the site is taken to be 2 m, then the limiting period is T_P = 4.42 s. For the deep water method, the largest period is calculated to be 1.66 s for these wind speeds. Therefore, no correction for period has to be made. For wind speeds 23-26 m/s for the longest fetch, the calculated wave heights were greater than 0.6*d (or 1.2 m), so the wave heights were adjusted down to 1.2 m. Just as a test, a wind speed of

26 m/s and a fetch of 10 km were tried. The calculated period is 4.01 s, so still under the limiting period for a 2 m depth. In these cases, if the site has one or two long fetches which may have some large waves during strong wind events, statistically those waves will not be weighted heavily. The question then becomes one of which method is better for areas that have long fetches in many directions. If the largest wave heights are of interest, then this should be examined further.

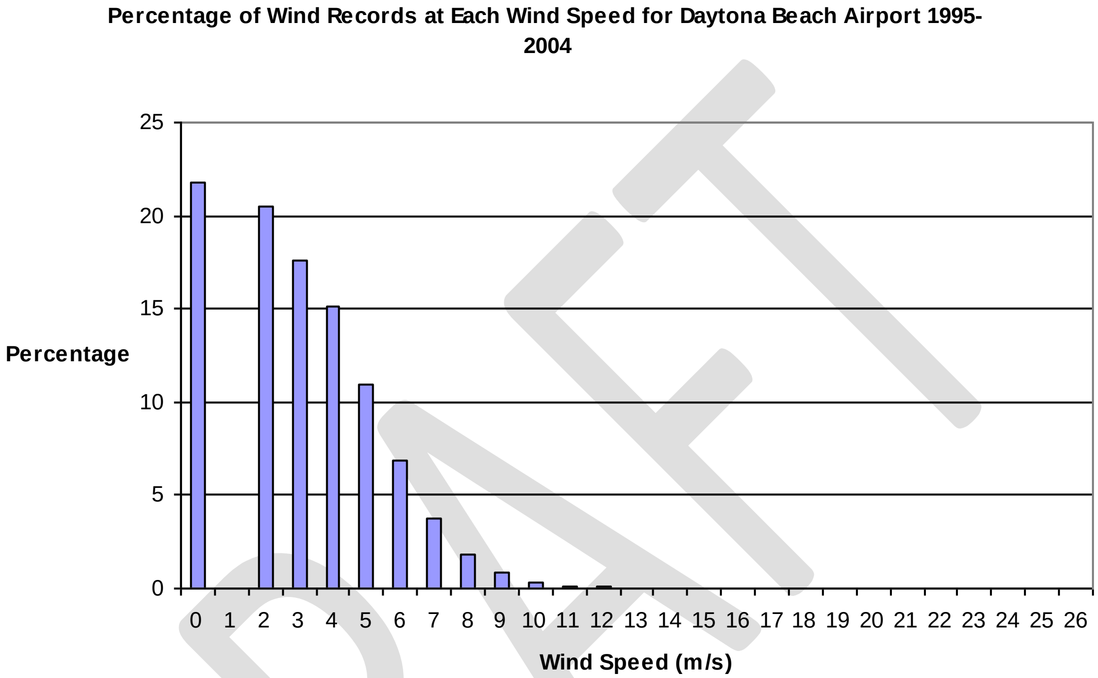

Figure 6: Percentage of wind records at each wind speed for Daytona Beach Airport for June 1995- December 2004.

Wave Height Thresholds for Marshes

Roland and Douglass (2005) studied sites around Alabama. Using a similar method to the one outlined here, they calculated the wave heights at sites with and without *Spartina alterniflora*. They found *S. alterniflora* was located at sites which had a 50th percentile wave for a long term median of less than 0.13 m and an 80th percentile wave of less than 0.20 m. They referenced a rule of thumb that the critical wave height for the 95th to 99th percentile wave is 1 ft (0.34 m). The waves falling into these percentiles for their study were between 0.30 and 0.40 m, which agrees with the rule of thumb.

Boat Wakes

Boat wakes also need to be considered along the shorelines, particularly in areas where the ICW has been cut directly through upland or marsh. Although it can be higher, the upper limit for boat wakes tends to come in around 0.6 m. Bhowmik et. al. (1990) (as cited in Maynord and Martin (1996)) studied sites along the Illinois and Mississippi Rivers on a Labor Day weekend. Data from controlled runs showed the generated boat wakes were mainly 0.14 m with a max of 0.58 m. Boater's Guides from Wisconsin and Oregon point out that runabouts and larger fishing boats can create a 10 in (0.254 m) wake, while houseboats and cruisers, which have displacement hulls, can create at least a 25 in (0.635 m) wake. Knutson et. al. (1990) gathered data on boats in the ICW in Swansboro, North Carolina. They were interested in a wind-sheltered dredge material island which had a salt marsh planted on it for stability. The island sees a fetch of only 0.5 km, but has 25000 boats pass by a year. They found most of the boat generated waves were 15 cm or less. They also found that even though this was a heavily traveled area, wind waves were 10 times more frequent than boat wake waves and that the waves from the boats were probably only responsible for less than 5% of the wave energy hitting the shore. From what I've seen, using a boat wake height of 0.60 m for calculations should cover it.

The waves decay with increasing distance from the boat. BoatWakes.info cites a few different studies for the equations to use for the decay. Stoker (1957) and Cox (2002) found that the maximum height of waves in a boat wake "decreases in proportion to the cube root of the distance from the boat."

$$\textit{Wave Height} = \textit{constant} * \textit{distance}^{-0.33} \qquad (17)$$

Cox (2002) says that in shallow water, the decrease in height of the highest wave may be a little more pronounced and suggests using

$$\textit{Wave Height} = \textit{constant} * \textit{distance}^{-0.4} \qquad (18)$$

Maynord (2001) studied boat waves on Johnson Lake and Kenai River, Alaska and found that wave decay for boat waves generated at maximum power for the boats decayed at $X^{-0.40}$ for Johnson Lake and $X^{-0.29}$ for the Kenai River.

Zabawa & Ostrom (1980) showed that while the maximum wave height does decrease with distance, the energy does not. Doctors (2002) says that the wave height itself does not represent "either the wave energy or the damage-causing ability of a wave system. The choice about how to characterize wave magnitude is not at all clear and noone has yet suggested a satisfactory answer to this question." According to BoatWakes.info, "A decrease in wave height is not necessarily a decrease in erosive energy, because the largest wave may be transferring its energy to adjacent waves."

The following pretty much comes from DHI (2004) to describe boat wakes.

The celerity of a gravity wave in shallow water is

$$C = L / T = \sqrt{gd} \quad (19)$$

where:
L = wavelength
T = wave period
d = depth

The Depth Based Froude Number is the ratio of the boat's speed (V) to the celerity (C).

$$Fd = V / C = V / \sqrt{gd} \quad (20)$$

where:
V = vessel speed

If Fd = 1, then the wave is going as fast as it can. If the boat goes faster than this, the wave will grow. The boat then needs enough power to climb up on the bow wave and get up on a plane DHI (2004). For deep water, the Vessel Length Froude Number is used.

$$Fl = V / \sqrt{gL_{BP}} \quad (21)$$

where:
L_{BP} = the length of the boat

Fl ≈0.4-0.55 is similar to Fd = 1 in shallow water.

For Fd<0.7, it is a sub-critical, deep water condition. Symmetrical sets of diverging waves come from the bow of the boat. Transverse waves propagate along the sailing line. These two types of waves meet along cusp locus lines which are at a 19^{o} 28' angle from the sailing line.

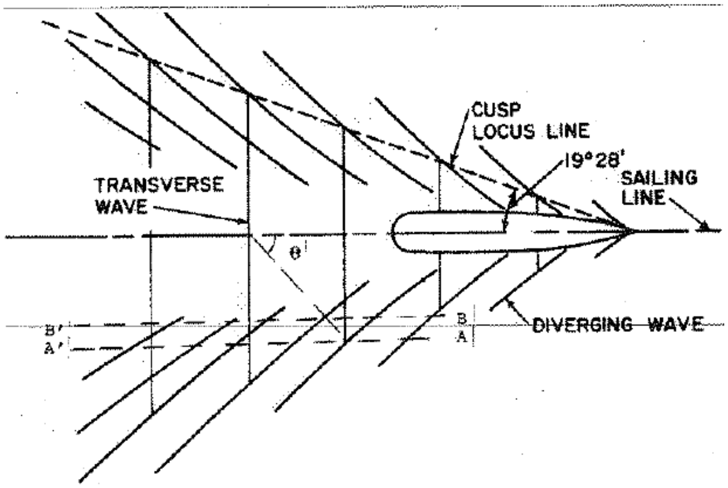

Figure 7: *Wave pattern generated by vessel moving at sub critical speed and water depth*

Taken from DHI (2004).

Transverse waves will travel at the same speed as the boat. The divergent waves will travel at a celerity of

$$C = V_s \cos\theta \quad (22)$$

$\theta = 35^o\ 16'$ for $Fd < 0.7$

When Fd exceeds 0.56, the transverse waves will start interacting with the bottom. At higher Fd's, the shorter divergent waves will also start interacting with the bottom. When Fd > 0.7, then the waves will start to significantly feel the bottom. Between 0.7<Fd<1.0, the transverse wave heights grow quicker than the divergent wave heights. The cusp locus angle increases until it is 90^o at Fd = 1. Along with that, θ decreases to 0^o. In equation form (from Weggel and Sorensen, 1986).

$$\theta = 35.27^o\left(1 - e^{12(F_d - 1)}\right) \quad (23)$$

for $0 \le Fd \le 1$.

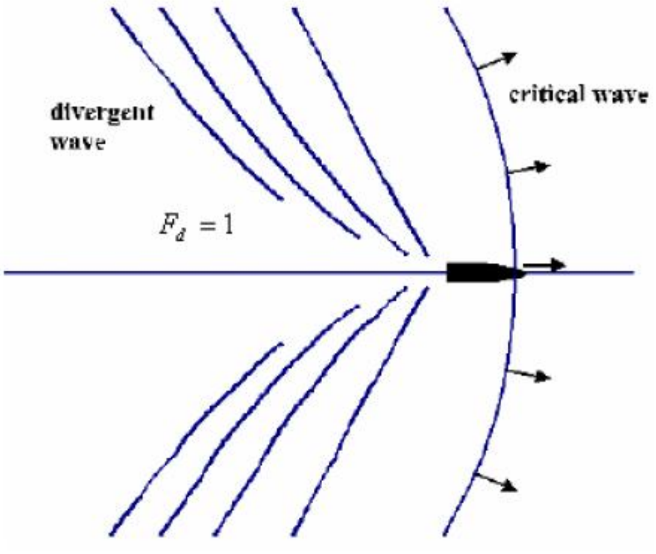

Figure 8: *Wave pattern at critical Froude number, Fd = 1*

Taken from DHI (2004).

While a critical value of Fd = 1 is assumed, it can vary with water depth. For deep water, the maximum wave energy can occur at a Fd value as low as 0.85. For shallow water it can occur at as much as Fd = 1.1.

In shallow water, we can see super-critical wash wave patterns for Fd > 1, for fast ferries (not applicable here) or planning speedboats (probably applicable here). In this case, the transverse waves can no longer keep up with the boat and so divergent waves are the only ones we have. You end up with the following wave pattern:

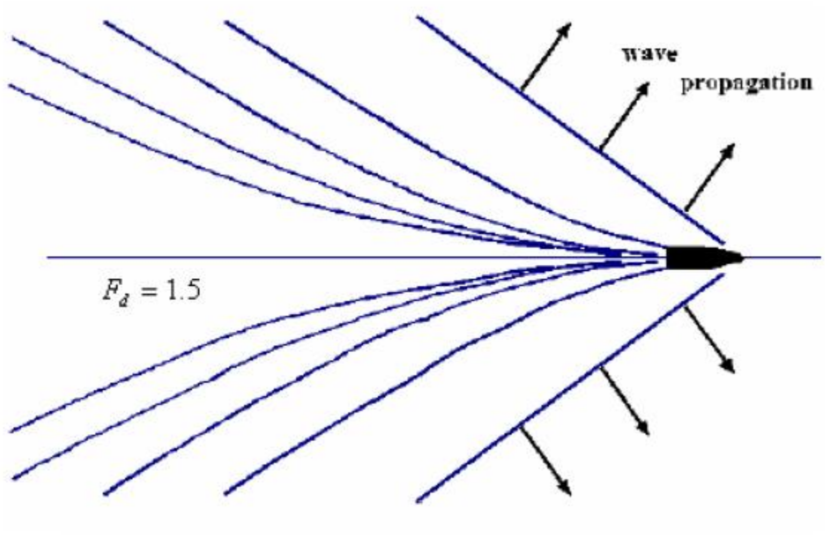

Figure 9: *Super critical wave wash pattern*

Taken from DHI (2004).

For Fd ≥1 , the angle θ is found from the equation

$$\theta = 90^o - \arccos\left(\frac{\sqrt{gH}}{V_s}\right) \quad (24)$$

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